## A First Course on Kinetics and Reaction Engineering

## Unit 34. 2-D and 3-D Tubular Reactor Models

## Definitions

radial dispersion coefficients - diffusivity-like and conductivity-like constants used to model movement of mass and heat in the radial direction

## Nomenclature

$\Delta H_{j} \quad$ heat of reaction $j$
$\alpha_{w} \quad$ wall heat transfer coefficient
$\left(\lambda_{e r}\right)_{s} \quad$ effective radial conductivity based on the superficial velocity
$v_{i, j} \quad$ stoichiometric coefficient of species $i$ in reaction $j$; negative for reactants and positive for products
$\rho_{\text {fluid }}$ density of the fluid
$A \quad$ cross-sectional area of the tube
$C_{i} \quad$ concentration of species $i$; a superscripted " 0 " denotes the value at the reactor inlet
$\tilde{C}_{p, \text { fluid }}$ mass-specific heat capacity of the fluid
$\left(D_{e r}\right)_{s} \quad$ effective radial diffusivity based on the superficial velocity
$G \quad$ mass velocity
Mi molecular weight of species $i$
$P \quad$ pressure; a superscripted " 0 " denotes the value at the reactor inlet
$R \quad$ tube radius
$T$ temperature; a superscripted " 0 " denotes the value at the reactor inlet, a subscripted " $w$ " denotes the wall temperature
$\dot{V} \quad$ volumetric flow rate
$d_{p} \quad$ particle diameter
$f$ friction factor
$\dot{m} \quad$ mass flow rate
$r \quad$ radial distance from the tube centerline
$r_{j} \quad$ rate of reaction $j$
$u \quad$ linear velocity; a subscripted $s$ denotes the superficial velocity
$z \quad$ axial distance from the reactor inlet

## Equations

$$
\begin{align*}
& D_{e r}\left(\frac{\partial^{2} C_{i}}{\partial r^{2}}+\frac{1}{r} \frac{\partial C_{i}}{\partial r}\right)-\frac{\partial}{\partial z}\left(u_{s} C_{i}\right)=\sum_{\substack{j=a l l \\
\text { reactions }}} v_{i, j} r_{j}  \tag{34.1}\\
& \lambda_{e r}\left(\frac{\partial^{2} T}{\partial r^{2}}+\frac{1}{r} \frac{\partial T}{\partial r}\right)-u_{s} \rho_{f \text { fuid }} \tilde{C}_{p, f l u i d} \frac{\partial T}{\partial z}=\sum_{\substack{j=\text { all } \\
\text { reacions }}} r_{j} \Delta H  \tag{34.2}\\
& -\frac{d P}{d z}=f \frac{\rho_{f l u i d} u_{s}^{2}}{d_{p}}  \tag{34.3}\\
& C_{i}(r, 0)=C_{i, \text { feed }}  \tag{34.4}\\
& T(r, 0)=T_{\text {feed }}  \tag{34.5}\\
& P(0)=P_{\text {feed }}  \tag{34.6}\\
& \left.\frac{\partial C_{i}}{\partial r}\right|_{r=0}=0  \tag{34.7}\\
& \left.\frac{\partial C_{i}}{\partial r}\right|_{r=R}=0  \tag{34.8}\\
& \left.\frac{\partial T}{\partial r}\right|_{r=0}=0  \tag{34.9}\\
& \left.\frac{\partial T}{\partial r}\right|_{r=R}=\frac{\alpha_{w}}{\lambda_{e r}}\left(T(R, z)-T_{w}\right)  \tag{34.10}\\
& u_{s} \rho_{\text {fluid }}=G \Rightarrow u_{s}=\frac{G}{\rho_{\text {fluid }}}  \tag{34.11}\\
& G=\frac{\dot{m}}{A}  \tag{34.12}\\
& \rho_{\text {fluid }}=C_{j} M_{j}+\sum_{i \neq j} C_{i} M_{i}  \tag{34.13}\\
& C_{j}=C_{\text {total }}-\sum_{i \neq j} C_{i}  \tag{34.14}\\
& C_{\text {total }}=\frac{P}{R T} \tag{34.15}
\end{align*}
$$

$$
\begin{equation*}
u_{s}=\frac{G}{\left(\frac{P}{R T}-\sum_{i \neq j} C_{i}\right) M_{j}+\sum_{i \neq j} C_{i} M_{i}} \tag{34.16}
\end{equation*}
$$

