A First Course on Kinetics and Reaction Engineering

Example 4.4

Problem Purpose

This example introduces one very common rate expression for cell growth, the Monod equation, and explores its features.

Problem Statement

The Monod equation, given below, can sometimes be used to model cell growth processes. In that equation, μ is the specific growth rate, C_S is the mass concentration of substrate, and μ_{max} and K_s are constants (they are sometimes referred to as the maximum specific growth rate and the saturation constant, respectively).

$$\mu = \frac{\mu_{\max} C_S}{K_s + C_S}$$

a. If μ_{max} = 1 h^{-1} and K_s = 0.2 g L^{-1}, plot μ vs C_S over a sufficiently wide range of concentration to show all limiting behavior
b. Describe any asymptotic behavior both mathematically and in terms of what is happening physically.

Problem Solution

(a) A series of values of C_S can be used to compute corresponding values of μ, and the results can be plotted. This is done in the Excel spreadsheet that accompanies this example to produce Figure 1 on the next page.

(b) As can be seen in the figure, at low concentrations the Monod equation predicts a linear dependence of the growth rate upon substrate concentration. This can be seen in the limit where C_S << K_s:

$$\mu = \frac{\mu_{\max} C_S}{K_s + C_S} \xrightarrow[C_S \ll K_s]{} \left(\frac{\mu_{\max}}{K_s}\right) C_S$$

In terms of a physical explanation, when the concentration of substrate (food) is very small, the cells must compete for it. As a result, the substrate is used very efficiently (if one cell doesn't quickly consume it, another cell does). Under these circumstances, the amount of substrate limits how fast the cells can grow; adding a little more substrate causes a proportional increase in the cell growth rate, because the cells are still competing for the substrate.

It can also be seen that the growth rate approaches a maximum value of 1 h^{-1} at very large substrate concentrations. This can again be seen in the limit where $C_S >> K_s$:

$$\mu = \frac{\mu_{\max} C_S}{K_s + C_S} \xrightarrow[C_S \gg K_s]{} \mu_{\max}$$

Continuing with the physical explanation, at these conditions there is an abundance of substrate. Each cell can have as much substrate as it wants; what limits the growth here is the number of cells present in the system. Thus, the rate of growth per cell or per cell mass (i. e. the specific cell growth rate) is constant.

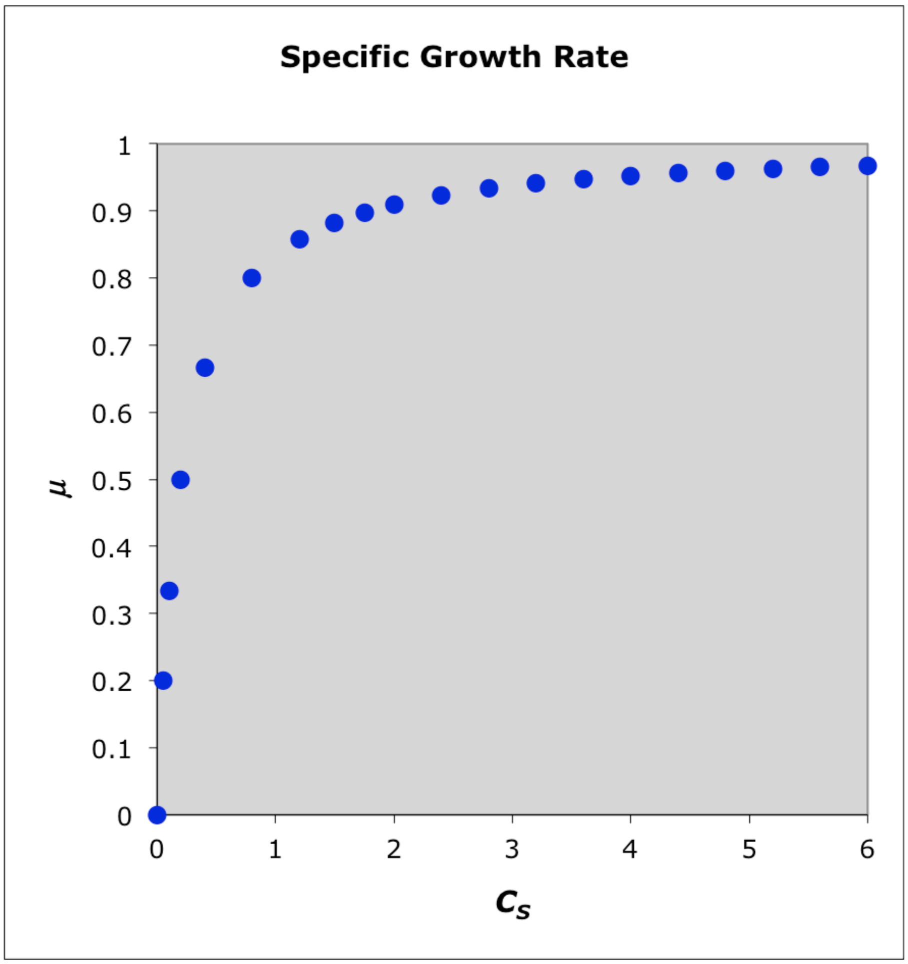

Figure 1. A plot of the specific cell growth rate versus substrate concentration as predicted by the Monod equation.